1. Calculate the square root of 24×150 (without first evaluating 24×150).

   Repeat for 147×48

   Repeat for $abc^2 \times c^4ab$

2. Prove that if n is an integer then
$$n^3 - n$$
   is divisible by 3 .

3. [2008 Oxford Admission Test question 1E] (Multiple Choice)

   The highest power of x in
$$f(x) = \left\{ \left[\left(2x^6 + 7\right)^3 + \left(3x^8 - 12\right)^4 \right]^5 + \left[\left(3x^5 - 12x^2\right)^5 + \left(x^7 + 6\right)^4 \right]^6 \right\}^3$$
   is

   (a) x^{424} (b) x^{450} (c) x^{500} (d) x^{504}

4. [2004 STEP I question 1]

   (a) Express $(3 + 2\sqrt{5})^3$ in the form $(a + b\sqrt{5})$ where a and b are integers.

   (b) Find the positive integers c and d such that $\sqrt[3]{99 - 70\sqrt{2}} = c - d\sqrt{2}$.

   (c) Find the two real solutions of $x^6 - 198x^3 + 1 = 0$.

5. [2008 Oxford Admission Test question 2]

   (a) Find a pair of positive integers x_1 and y_1, that solve the equation

   $$(x_1)^2 - 2(y_1)^2 = 1\,.$$

   (b) Given integers a and b we define sequences $x_1, x_2, x_3, \ldots$ and $y_1, y_2, y_3, \ldots$ by setting

   $$x_{n+1} = 3x_n + 4y_n \quad \text{and} \quad y_{n+1} = ax_n + by_n \quad \text{for} \quad n \geq 1\,.$$

   Find *positive* values of a and b such that

   $$(x_n)^2 - 2(y_n)^2 = (x_{n+1})^2 - 2(y_{n+1})^2\,.$$

   (c) Find a pair of integers X, Y which satisfy $X^2 - 2Y^2 = 1$ such that $X > Y > 50$.

   (d) (Using the values of a and b found in part (b).) What is the approximate value of $\dfrac{x_n}{y_n}$ as n increases?

6. [2004 STEP I question 3]

   (a) Show that $x - 3$ is a factor of

   $$x^3 - 5x^2 + 2x^2y + xy^2 - 8xy - 3y^2 + 6x + 6y\,. \qquad (*)$$

   Express (*) in the form $(x-3)(x+ay+b)(x+cy+d)$ where a, b, c and d are integers to be determined.

   (b) Factorise $6y^3 - y^2 - 21y + 2x^2 + 12x - 4xy + x^2y - 5xy^2 + 10$ into three linear factors.

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